MATH-5C TEST 3 Sample
(Chapter 15, 16.2i, 16.7i)

100 points NAME:__________________

Be sure to show work neatly and follow instructions carefully.

Longer than actual exam

(1) Evaluate $\iint\limits_R \dfrac{xy}{\sqrt{x^2+y^2+1}}\,dA \quad R:\{(x,y):0 \le x \le 1,\ 0 \le y \le 1\}$.

Ans: $\frac{1}{3}-\frac{4\sqrt{2}}{3}+\sqrt{3}$

(2) SET UP BUT DO NOT EVALUATE: Use a double integral in polar coordinates to find the volume of the solid bounded above by the surface $z = 1 - x^2 - y^2$, below by the xy plane, and laterally by the cylinder $x^2 + y^2 - x = 0$.

Ans: $\displaystyle\int_{-\pi/2}^{\pi/2}\int_0^{\cos\theta}(1-r^2)r\,dr\,d\theta$

(3) Evaluate $\displaystyle\int_0^2\int_{y/2}^1 \cos(x^2)\,dx\,dy$

Ans: $\sin 1$

(4)

(a) Convert $(4, \pi/2, 3)$ from cylindrical coordinates to

rectangular coordinates ________________ Ans: (0,4,3)

spherical coordinates __________________ Ans: $(5, \pi/2, \cos^{-1}(3/5))$

(b) Convert $(2, 2, \sqrt{2})$ from rectangular coordinates to

cylindrical coordinates ________________ Ans: $(2\sqrt{2}, \pi/4, \sqrt{2})$

spherical coordinates __________________Ans: $(\sqrt{10}, \pi/4, \cos^{-1}(1/\sqrt{5}))$

(5) SET UP BUT DO NOT EVALUATE: $\iiint\limits_E f(x,y,z)\,dV$ where E is the solid bounded by the paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $z = 4 - 3y^2$.

Ans: $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{4x^2+y^2}^{4-3y^2} f(x,y,z)\,dz\,dy\,dx$ (many others possible)

(6) SET UP BUT DO NOT EVALUATE: integrals as specified to find the volume enclosed by the cone $z^2 = x^2 + y^2$ and the plane z = 4.

a) Triple integral - cylindrical coordinates.

Ans: $\int_0^{2\pi}\int_0^4\int_r^4 r\, dz dr d\theta$

b) Triple integral - spherical coordinates.

Ans: $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{4\sec\phi} \rho^2 \sin\phi\, d\rho d\varphi d\theta$

c) Double integral - rectangular coordinates ; <u>order dy dx</u> .

Ans: see solutions

d) Triple integral- rectangular coordinates; <u>order dx dz dy</u>

Ans:

(7) Given $\int_0^{2\pi}\int_0^1\int_0^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta$

a) The above integral can be used to compute the volume of a solid. Sketch the solid.

Ans: It is a cylinder of radius one topped by a portion of a sphere of radius 2.

b) Convert the triple integral to <u>Rectangular Coordinates</u>: DO NOT EVALUATE.

Ans. $4\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{4-x^2-y^2}} dz dy dx$

c) Convert the triple integral to <u>Spherical Coordinates</u>: DO NOT EVALUATE.

Ans: This one is a little difficult, must be split into two solids

$$\int_0^{2\pi}\int_0^{\pi/6}\int_0^2 \rho^2 \sin\phi\, d\rho d\phi d\theta + \int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^{\csc\phi} \rho^2 \sin\phi\, d\rho d\phi d\theta$$

(8) Evaluate $\iint_S z\, dS$ where S is the portion of the paraboloid $z=x^2+y^2$ that lies under the plane $z=4$. Ans: $\frac{\pi}{60}\left(391\sqrt{17}+1\right)$

(9) A lamina the shape of a quarter circle of radius 4 has density proportional to the distance from the origin. Find the center of mass. Ans: $\left(\frac{6}{\pi},\frac{6}{\pi}\right)$

(10) $\int_C yz\cos x\, ds$ where C is given by x=t, y=2cost, z=2sint, $0 \le t \le \pi$. Ans: $\frac{8\sqrt{5}}{3}$